***Circular Motion***

* Motion in a circle at a constant “speed” is still accelerating.
* As the direction changes, the acceleration affects the velocity vector. The magnitude of the velocity is unaffected.
* A point mass “m” with a constant magnitude of velocity “|v|” travelling in a circle of radius “r” undergoes acceleration.
* The acceleration is called the centripetal acceleration and it acts towards the centre of the circle.
* The velocity vector is tangential to the path of rotation. (A rotating object released would follow a straight line at the point of release).
* Newton’s 1st law relates to inertia, objects have a tendency to travel in a straight line. Newton’s 2nd law requires a net force for acceleration.
* The position vector r and the velocity vector v rotate at the same rate.
* A fractional change in any time is the same for a constant acceleration.

r

v

v + Δv

r + Δr

θ

r + Δr

r

Δr

θ

v

v + Δv

Δv

θ

* The vector diagrams show r + Δr and v + Δv and both have the same angle θ.

|Δv| = |Δr|

|v| |r|

Dividing both sides by Δt gives;

1|Δv| = 1|Δr|

v|Δt| r|Δt|

As Δr/Δt = velocity and Δv/Δt = acceleration:

a = v

v r

ac = v2

r

* Distance travelled for a circle is 2πr and the time for a rotation is called the period, T.
* The speed can then be found using v = Δd/Δt which becomes v = 2πr/T.
* Frequency (ƒ) is the # of rotations per second, measured in Hertz (Hz) and ƒ= 1/T
* The centripetal force Fc = mac, so Fc = mv2

r

* The centripetal force is supplied by some external source. The attraction of the Earth to the sun maintains its elliptical path so gravity supplies the centripetal force.
* Friction supplies Fc for a car going around a curve.
* Horizontal rotations are the easiest to study as we can ignore gravity (independent motion) assuming the object is level. (FT = Fc)

***Examples:***

1. A car turns a level circular curve with a speed of 20.0 m/s through a radius of 100.0 m. What is the centripetal acceleration?
2. Find the centripetal acceleration and force for a 10.0 kg stone whirled in a horizontal circle at the end of a 1.50 m string with a frequency of 1.25 Hz.

***Vertical Circular Motion***

* Vertical rotations have varying centripetal force at each point of the rotation. (Tension forces in ropes will therefore vary).
* The easiest method to solve these problems is to first draw an FBD of the forces involved.
* Treat Fc as the net force (the net force supplies Fc) and determine the vector solution that satisfies this result.
* We will only solve problems for the top or bottom of a vertical spin (linear forces).
* The diagram shows a biologist’s head whirled on the end of a string. (The mass of the string is negligible, like the amount of brains in the head).

Fg

Fg

Fc

Fc

FT

FT

Top

Fc is down, so Fnet is too

Fc = FT + Fg

Bottom

Fc is up, so Fnet is too FT must be > Fg to make Fnet up

Fc = FT - Fg

Examples

1. An object swings vertically at a constant radius of 0.75 m. What is the minimum speed at the top of the rotation?

FT = 0 for minimum speed (just stays circular)

Fc = Fg

mv2/r = mg

v = (rg)½

No direction as questions asks for speed!

1. A 1.7 kg object is swung vertically at the end of a 0.60 m string. If the period is 1.1 s, what’s the tension at the top and bottom of the rotation?

Answer: 17 N [S] and 50. N [N]

1. A 1200.0 kg car drives at 13.0 m/s down a circular hill which has a radius of 150.0 m. Find the apparent weight of the car at the bottom of the hill.

Fg

FN

Fc

Fc = FN – Fg

FN is the apparent weight so FN = Fc + Fg

**Banked Curves**

* Curves off highways are curved to minimize the amount of sliding.
* A component of the normal force supplies some of the centripetal force (along with friction) such that there is an angle of incline where no friction is needed.
* Drawing an FBD will aid us in developing the relationship to calculate the angle.

Fg

Fc

FN

θ

Fg + FN = Fc

θ

Fg

FN

Fc

Recall that Fc is supplied by the net force, so the vector sum of Fg and FN give Fc in the diagram.

tan θ = Fc/Fg

= mv2/mgr

v2 = rgtanθ

Note: This formula must be derived.

Example:

1) Find the speed for a 80.0 m radius curve banked at 20.0o to maintain non-skidding travel.